Class X Session 2023-24
Subject - Mathematics (Basic)
Sample Question Paper - 6

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A, B, C, D and E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with sub- parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

Section A

1. If $a = 2^3 \times 3, b = 2 \times 3 \times 5, c = 3^n \times 5$ and LCM (a, b, c) $= 2^3 \times 3^2 \times 5$, then n = **[1]**

a) 1 b) 4

c) 3 d) 2

2. The sum of the exponents of the prime factors in the prime factorisation of 196, is **[1]**

a) 2 b) 1

c) 4 d) 6

3. One of the two students, while solving a quadratic equation in x, copied the constant term incorrectly and got the roots 3 and 2. The other copied the constant term and coefficient of x^2 correctly as -6 and 1 respectively. The correct roots are _________. **[1]**

a) 6, -1 b) 3, -2

c) -3, 2 d) -6, -1

4. The graphs of the equations 5x - 15y = 8 and 3x - 9y = $\frac{24}{5}$ are two lines which are **[1]**

a) intersecting exactly at one point b) coincident

c) perpendicular to each other d) parallel

5. The quadratic equation, sum of whose roots is $3\sqrt{2}$ and their product is 5, is **[1]**

a) $x^2 + 3\sqrt{2}x - 5 = 0$ b) $x^2 + 3\sqrt{2}x + 5 = 0$

c) $x^2 - 3\sqrt{2}x - 5 = 0$ d) $x^2 - 3\sqrt{2}x + 5 = 0$

6. The length of the median through A of △ABC with vertices A(7, -3), B(5, 3) and C(3, -1) is **[1]**

a) 5 units b) 3 units

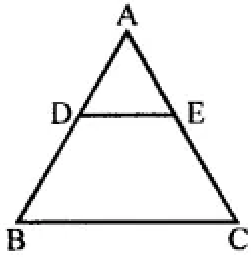
c) 7 units d) 25 units

7. In a △ABC, AD is the bisector of ∠BAC. If AB = 8cm, BD = 6cm and DC = 3 cm. Find AC **[1]**

a) 6 cm b) 4 cm

c) 3 cm d) 8 cm

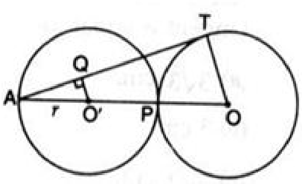
8. In the given figure, DE||BC. If DE = 5 cm, BC = 8 cm and AD = 3.5 cm then AB = ? **[1]**



a) 4.8 cm b) 5.6 cm

c) 5.2 cm d) 6.4 cm

9. Equal circles with centre O and O' touch each other at P. O and Q' touch each other at P. OO' is produced to meet circle (O', r) at A. AT is a tangent to the circle (O, r). O' Q is perpendicular to AT. Then the value of $\frac{AO'}{AO}$ is **[1]**



a) $\frac{1}{2}$ b) $\frac{2}{3}$

c) $\frac{1}{4}$ d) $\frac{1}{3}$

10. If $\sin A + \sin^2 A = 1$, then the value of the expression $(\cos^2 A + \cos^4 A)$ is **[1]**

a) $\frac{1}{2}$ b) 1

c) 3 d) 2

11. From a lighthouse, the angles of depression of two ships on opposite sides of the lighthouse are observed to be 30° and 45°. If the height of the lighthouse is h meters, the distance between the ships is **[1]**

a) $1 + \left(1 + \frac{1}{\sqrt{3}}\right)$ h metres b) $\sqrt{3}$ h metres

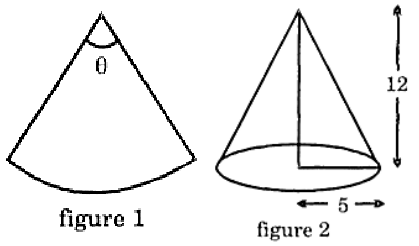
c) $(\sqrt{3} + 1)$ h metres d) $(\sqrt{3} - 1)$ h metres

12. If $a \cos\theta + b \sin\theta = m$ and $a \sin\theta - b \cos\theta = n$, then $a^2 + b^2 =$ **[1]**

a) $n^2 - m^2$ b) $m^2 + n^2$

c) $m^2 - n^2$ d) m^2n^2

13. A piece of paper in the shape of a sector of a circle (see figure 1) is rolled up to form a right-circular cone (see figure 2). The value of angle θ is: **[1]**



a) $\frac{5\pi}{13}$

b) $\frac{6\pi}{13}$

c) $\frac{10\pi}{13}$

d) $\frac{9\pi}{13}$

14. In a circle of radius 14 cm, an arc subtends an angle of 120^0 at the centre. If $\sqrt{3}$= 1.73 then the area of the segment of the circle is **[1]**

a) 124.63 cm^2

b) 130.57 cm^2

c) 120.56 cm^2

d) 118.24 cm^2

15. A bag contains 8 red, 2 black and 5 white balls. One ball is drawn at random. What is the probability that the ball drawn is not black? **[1]**

a) $\frac{13}{15}$

b) $\frac{1}{3}$

c) $\frac{8}{15}$

d) $\frac{2}{15}$

16. The mean of 2, 7, 6 and x is 5 and the mean of 18, 1, 6, x and y is 10. What is the value of y? **[1]**

a) 30

b) 10

c) 5

d) 20

17. A cubical block of side 7 cm is surmounted by a hemisphere. The greatest diameter of the hemisphere is **[1]**

a) 10.5cm

b) 7cm

c) 3.5cm

d) 14cm

18. Which of the following cannot be determined graphically? **[1]**

a) Mode

b) Median

c) Mean

d) None of these

19. **Assertion (A):** Ratio in which the line 3x + 4y = 7 divides the line segment joining the points (1, 2) and (-2, 1) is 3: 5 **[1]**

Reason (R): The coordinates of the point P(x, y) which divides the line segment joining the points A(x_1, y_1) and B(x_2, y_2) in the ratio m_1: m_2 is $\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** H.C.F. of 12 and 77 is 1. **[1]**

Reason (R): L.C.M. of two coprime numbers is equal to their product.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

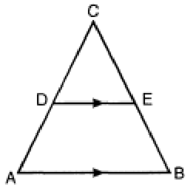
c) A is true but R is false.

d) A is false but R is true.

Section B

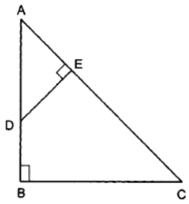
21. Two rails are represented by the equations: x + 2y – 4 = 0 and 2x + 4y – 12 = 0. Will the rails cross each other? **[2]**

22. In the given figure, $\angle A = \angle B$ and AD = BE. Show that DE $\|$AB. **[2]**

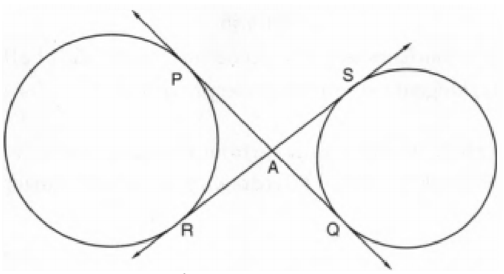


OR

In Fig. if $AB \perp BC$ and $DE \perp AC$. Prove that $\triangle$ABC ~ $\triangle$AED.



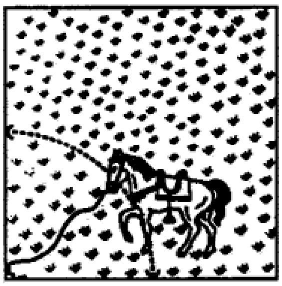
23. In fig common tangents PQ and RS to two circles intersect at A. Prove that PQ = RS. **[2]**



24. Evaluate : $\frac{\cos 45^\circ}{\sec 30^\circ} + \frac{1}{\sec 60^\circ}$. **[2]**

25. A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope. Find **[2]**

   i. the area of that part of the field in which the horse can graze.
   ii. the increase in the grazing area if the rope were 10 m long instead of 5 m (Use π = 3.14)



OR

A piece of wire 20 cm long is bent into the form of an arc of a circle subtending an angle of 60° at its centre. Find the radius of the circle.

Section C

26. Explain why 7 × 11 × 13 + 13 and 7 × 6 × 5 × 4 × 3 × 2 × 1 + 5 are composite numbers. **[3]**

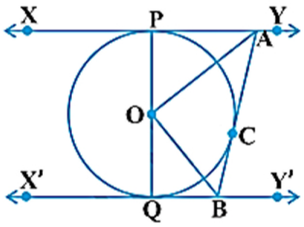
27. If α and β are zeroes of the quadratic polynomial $4x^2 + 4x + 1$, then form a quadratic polynomial whose zeroes are 2α and 2β. **[3]**

28. Solve 2x + 3y = 11 and 2x – 4y = –24 and hence find the value of **m** for which y = mx + 3. **[3]**

OR

Aditya is walking along the line joining points (1,4) and (0,6). Aditi is walking along the line joining points (3,4) and (1,0). Represent the graph and find the point where both cross each other.

29. In Figure, XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersects XY at A and X'Y' at B. Prove that $\angle AOB$ = 90°. **[3]**



30. Prove that: $(1+\tan^2 A)+\left(1+\frac{1}{\tan^2 A}\right)=\frac{1}{\sin^2 A-\sin^4 A}$ **[3]**

OR

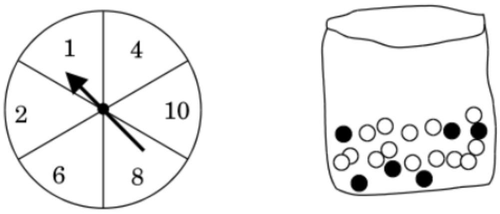
Prove that: sec A (1 - sin A) (sec A + tan A) = 1

31. Read the following passage and answer the questions given at the end: **[3]**

Diwali Fair

A game in a booth at a Diwali Fair involves using a spinner first. Then, if the spinner stops on an even number, the player is allowed to pick a marble from a bag. The spinner and the marbles in the bag are represented in Figure.

Prizes are given when a black marble is picked. Shweta plays the game once.



i. What is the probability that she will be allowed to pick a marble from the bag?

ii. Suppose she is allowed to pick a marble from the bag, what is the probability of getting a prize, when it is given that the bag contains 20 marbles out of which 6 are black?

Section D

32. If the price of a book is reduced by ₹5, a person can buy 5 more books for ₹ 300. Find the original list price of the book. **[5]**

OR

The perimeter of a rectangular field is 82 m and its area is 400 square metre. Find the length and breadth of the rectangle.

33. PQRS is a trapezium with PQ || SR Diagonals PR and SQ intersect at M and $\Delta PMS \sim \Delta QMR$. Prove that PS = QR. **[5]**

34. A toy is in the form of a cone mounted on a hemisphere of radius 3.5 cm. The total height of the toy is 15.5 cm; find the total surface area and volume of the toy. **[5]**

OR

A rocket is in the form of a right circular cylinder closed at the lower end and surmounted by a cone with the same radius as that of cylinder. The diameter and height of cylinder are 6 cm and 12 cm, respectively. If the slant height of the conical portion is 5 cm, then find the total surface area and volume of rocket. (Use π = 3.14)

35. In a retail market, fruit vendors were selling mangoes kept in packing boxes. These boxes contained varying number of mangoes. The following was the distribution of mangoes according to the number of boxes. **[5]**

Number of mangoes	50-52	53-55	56-58	59-61	62-64
Number of boxes	15	110	135	115	25

Find the mean number of mangoes kept in a packing box. Which method of finding the mean did you choose?

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Your friend Varun wants to participate in a 200m race. He can currently run that distance in 51 seconds and with each day of practice it takes him 2 seconds less. He wants to do in 31 seconds.

(i) Write first four terms are in AP for the given situations.

(ii) What is the minimum number of days he needs to practice till his goal is achieved?

OR

Out of 41, 30, 37 and 39 which term is not in the AP of the above given situation?

(iii) How many second takes after 5^{th} days?

37. **Read the text carefully and answer the questions:** **[4]**

The Chief Minister of Delhi launched the, 'Switch Delhi', an electric vehicle mass awareness campaign in the National Capital. The government has also issued tenders for setting up 100 charging stations across the city. Each station will have five charging points. For demo charging station is set up along a straight line and has charging points at $A\left(\frac{-7}{3}, 0\right)$, $B\left(0, \frac{7}{4}\right)$, C(3, 4), D(7, 7) and E(x, y). Also, the distance between C and E is 10 units.

(i) What is the distance DE?

(ii) What is the value of x + y?

OR

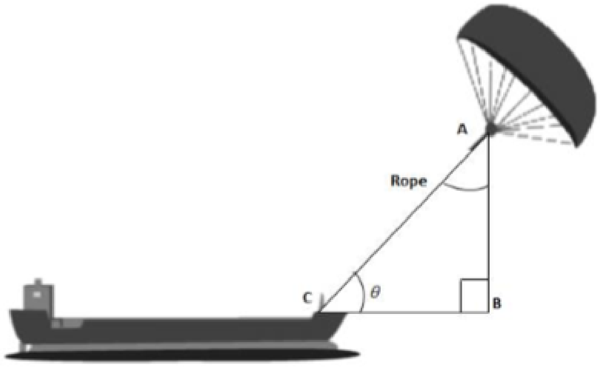
Points C, D, E are collinear or not?

(iii) What is the ratio in which B divides AC?

38. **Read the text carefully and answer the questions:** **[4]**

Skysails is the genre of engineering science that uses extensive utilization of wind energy to move a vessel in the seawater. The 'Skysails' technology allows the towing kite to gain a height of anything between 100 metres - 300 metres. The sailing kite is made in such a way that it can be raised to its proper elevation and then brought back with the help of a 'telescopic mast' that enables the kite to be raised properly and effectively.

Based on the following figure related to sky sailing, answer the following questions:



(i) In the given figure, if $\sin\theta = \cos(\theta - 30^{o})$, where θ and $\theta - 30^{o}$ are acute angles, then find the value of θ.

(ii) What should be the length of the rope of the kite sail in order to pull the ship at the angle θ (calculated above) and be at a vertical height of 200m?

OR

What should be the length of the rope of the kite sail in order to pull the ship at the angle θ (calculated above) and be at a vertical height of 150m?

(iii) In the given figure, if $\sin\theta = \cos(3\theta - 30^{o})$, where θ and $3\theta - 30^{o}$ are acute angles, then find the value of θ.

Solution

Section A

1.

(d) 2

Explanation: LCM (a, b, c) $= 2^3 \times 3^2 \times 5$ .... (I)

we have to find the value of n

Also we have

$a = 2^3 \times 3$

$b = 2 \times 3 \times 5$

$c = 3^n \times 5$

We know that the while evaluating LCM, we take greater exponent of the prime numbers in the factorisation of the number.

Therefore, by applying this rule and taking $n \geq 1$ we get the LCM as

LCM (a, b, c) = $2^3 \times 3^n \times 5$ ..... (II)

On comparing (I) and (II) sides, we get:

$2^3 \times 3^2 \times 5 = 2^3 \times 3^n \times 5$

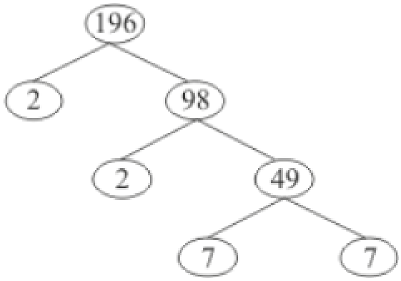
n = 2

2.

(c) 4

Explanation:

Using the factor tree for prime factorisation, we have:



Therefore,

$196 = 2 \times 2 \times 7 \times 7$

$196 = 2^2 \times 7^2$

The exponents of 2 and 7 are 2 and 2 respectively.

Thus the sum of the exponents is 4.

3. **(a)** 6, -1

Explanation: Let the equation be $x^2 + ax + b = 0$

Its roots are 3 and 2

$\therefore$Sum of roots, 5 = -a

and product of roots, 6 = b

$\therefore$Equation is $x^2 - 5x + 6 = 0$

Now constant term is wrong and it is given that correct constant term is -6.

$\therefore x^2 - 5x - 6 = 0$ is the correct equation.

Its roots are -1 and 6.

4.

(b) coincident

Explanation: We have,

5x - 15y - 8 = 0

And, 3x - 9y - $\frac{24}{5}$ = 0

Here, $a_1 = 5$, $b_1 = -15$ and $c_1 = -8$

And, $a_2 = 3$, $b_2 = -9$ and $c_2 = \frac{-24}{5}$

$\therefore \frac{a_1}{a_2} = \frac{5}{3}, \frac{b_1}{b_2} = \frac{-15}{-9} = \frac{5}{3}$ and $\frac{c_1}{c_2} = -8 \times \frac{5}{-24} = \frac{5}{3}$

Clearly, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Hence, the given system has a unique solution and the lines are coincident.

5.

(d) $x^2 - 3\sqrt{2}x + 5 = 0$

Explanation: Given: Sum of roots $(\alpha + \beta) = 3\sqrt{2}$ and Product of roots $(\alpha\beta) = 5$

$\therefore x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$\Rightarrow x^2 - 3\sqrt{2}x + 5 = 0$

6. **(a)** 5 units

Explanation: ABC is a triangle with A(7, - 3), B(5, 3) and C(3, - 1)

Let median on BC bisects BC at D. (AD is given as the median)

$\therefore$ Coordinates of D are $\left(\frac{5+3}{2}, \frac{3-1}{2}\right)$ = (4, 1)

$\therefore$ AD = $\sqrt{(4-7)^2 + (1+3)^2}$

= $\sqrt{9 + 16}$

= $\sqrt{25}$ = 5 units

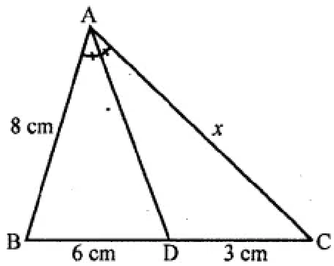
7.

(b) 4 cm

Explanation:

In $\triangle$ABC, AD is the bisector of $\angle$BAC

AB = 8 cm, BD = 6 cm and DC = 3 cm



Let AC = x

$\because$ In $\triangle$ ABC, AD is the bisector of $\angle$A

$\therefore \frac{AB}{AC} = \frac{BD}{DC} \Rightarrow \frac{8}{x} = \frac{6}{3}$

$\Rightarrow x = \frac{8 \times 3}{6} = 4$

$\therefore$ AC $=$ 4cm

8.

(b) 5.6 cm

Explanation: $\because$ DE $\parallel$ BC

$\therefore \frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$ (Thales' theorem)

$\Rightarrow \frac{3.5}{AB} = \frac{5}{8}$

$\Rightarrow AB = \frac{3.5 \times 8}{5}$ =5.6 cm

9.

(d) $\frac{1}{3}$

Explanation: $\frac{AO'}{AO} = \frac{r}{AO' + O'P + OP}$

= $\frac{r}{r+r+r}$ (Radii of both circles are equal)

$\Rightarrow \frac{AO'}{AO} = \frac{r}{3r} = \frac{1}{3}$

10.

(b) 1

Explanation: Given that, $\sin A + \sin^2 A = 1$

$\Rightarrow \sin A = 1 - \sin^2 A$

$\Rightarrow \sin A = \cos^2 A$

$\Rightarrow \sin^2 A = \cos^4 A$

$\Rightarrow 1 - \cos^2 A = \cos^4 A$

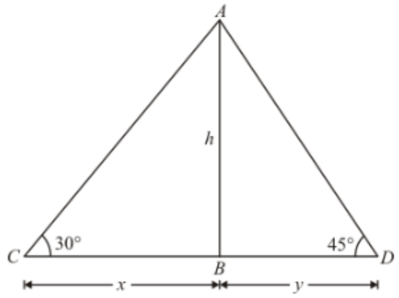
$\Rightarrow \cos^2 A + \cos^4 A = 1$

11.

(c) $(\sqrt{3} + 1)$ h metres

Explanation: Let the height of the light house AB be h meters

Given that: angle of depression of ship are $\angle C = 30^o$ and $\angle D = 45^o$



Distance of the ship C = BC = x and distance of the ship D = BD = y

Here, we have to find distance between the ships.

So we use trigonometric ratios.

In a triangle ABC,

$\Rightarrow \tan C = \frac{AB}{BC}$

$\Rightarrow \tan 30^\circ = \frac{h}{x}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x}$

$\Rightarrow x = \sqrt{3}h$

Again in a triangle ABD,

$\tan D = \frac{AB}{BD}$

$\Rightarrow \tan 45^o = \frac{h}{y}$

$\Rightarrow 1 = \frac{h}{y}$

$\Rightarrow y = h$

Now, distance between the ships = $x + y = \sqrt{3}h + h = (\sqrt{3} + 1)$ h

12.

(b) $m^2 + n^2$

Explanation: Given,

$a \cos\theta + b \sin\theta = m$

$a \sin\theta - b \cos\theta = n$

Now, Squaring and adding, we have;

$a^2 \cos^2\theta + b^2 \sin^2\theta + 2ab \sin\theta \cos\theta = m^2$

$a^2 \sin^2\theta + b^2 \cos^2\theta - 2ab \sin\theta \cos\theta = n^2$

$a^2 (\cos^2\theta + \sin^2\theta) + b^2 (\sin^2\theta + \cos^2\theta) = m^2 + n^2$ $\{\because \sin^2\theta + \cos^2\theta = 1\}$

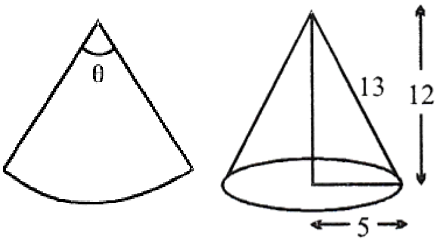
$\Rightarrow a^2 + 1 + b^2 \times 1 = m^2 + n^2$

$\Rightarrow a^2 + b^2 = m^2 + n^2$

Hence $a^2 + b^2 = m^2 + n^2$

13.

(c) $\frac{10\pi}{13}$

Explanation: 

$\therefore$ Slant height = 13

As, $\theta = \frac{S}{r}$

$\Rightarrow$ S = rθ

$\Rightarrow$ $2\pi(5) = 13\,\theta$

$\Rightarrow \theta = \frac{10\pi}{13}$

14.

(c) 120.56 cm^2

Explanation: ar(segment) = $\left(\frac{\pi r^2\theta}{360} - r^2 \sin\frac{\theta}{2}\cos\frac{\theta}{2}\right)$

$= \left(\frac{22}{7} \times 14 \times 14 \times \frac{120}{360}\right) - (14 \times 14 \times \sin 60^\circ \cos 60^\circ)$

$= \left(\frac{616}{3} - \frac{\sqrt{3}}{2} \times \frac{1}{2} \times 14 \times 14\right)\ \mathrm{cm}^2$

=(205.33 -49$\times$1.73) cm^2

=(205.33 - 84.77) cm^2

= 120.56 cm^2

15. **(a)** $\frac{13}{15}$

Explanation: Total number of balls in the bag = 8 + 2 + 5 = 15.

Number of non-black balls = 8 + 5 = 13.

$\therefore$ P (getting a non-black ball) = $\frac{13}{15}$

16.

(d) 20

Explanation: Mean of 2, 7, 6 and x = 5

$\Rightarrow \frac{2+7+6+x}{4} = 5$

$\Rightarrow$ 15 + x = 20

$\Rightarrow$ x = 5

Also, Mean of 18, 1, 6, x and y = 10

$\Rightarrow \frac{18+1+6+x+y}{5} = 10$

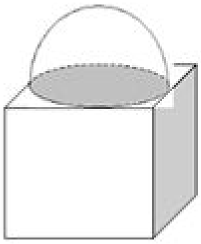
$\Rightarrow \frac{18+1+6+5+y}{5} = 10$

$\Rightarrow$ 30 + y = 50

$\Rightarrow$ y = 20

17.

(b) 7cm

Explanation: 

It is clear that Maximum diameter of hemisphere can be the side of the cube.

$\therefore$ The greatest diameter of the hemisphere = 7 cm

18.

(c) Mean

Explanation: Mode is the value with the maximum frequency. Thus, it can be determined from the graph.

Median is the middle value of the data. Thus, it can be determined from the graph.

Mean is the ratio of sum of all data values and the total number of values. Thus, it cannot be determined by graphically.

19.

(d) A is false but R is true.

Explanation: We know that the coordinates of the point P(x, y) which divides the line segment joining the points A(x_1, y_1) and B(x_2, y_2) in the ratio $m_1 : m_2$ is $\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$

So, Reason is correct.

Let the ratio is k:1.Here, $x_1 = 1$, $y_1 = 2$, $x_2 = -2$, $y_2 = 1$, $m_1 = k$, $m_2 = 1$

Now x-coordinate = $\frac{m_1x_2+m_2x_1}{m_1+m_2} = \frac{(k\times -2)+(1\times 1)}{k+1} = \frac{-2k+1}{k+1}$

and y-coordinate = $\frac{m_1y_2+m_2y_1}{m_1+m_2} = \frac{(k\times 1)+(1\times 2)}{k+1} = \frac{k+2}{k+1}$

Now, −6k + 3 + 4k + 8 = 7k + 7 ⇒ 7k + 2k = 11 − 7 ⇒ 9k = 4 ⇒ k = $\frac{4}{9}$

So, the Assertion is not correct

20.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation: Yes 12 and 17 are coprime numbers and H.C.F. of coprimes is always 1.

Section B

21. The pair of linear equations are given as:

x + 2y – 4 = 0 ...(i)

2x + 4y – 12 = 0 ...(ii)

We express x in terms of y from equation (i), to get

x = 4 – 2y

Now, we substitute this value of x in equation (ii), to get

2(4 – 2y) + 4y – 12 = 0

i.e., 8 – 12 = 0

i.e., – 4 = 0

Which is a false statement. Therefore, the equations do not have a common solution. So, the two rails will not cross each other.

22. In △CAB, ∠A = ∠B(Given)

∴ AC = CB (By isosceles triangle property)

But, AD = BE (Given).....(i)

⇒ AC- CD = CB - BE

∴ CD = CE ....(ii)

Dividing equation (ii) by (i),

$\frac{CD}{AD} = \frac{CE}{BE}$

By converse of BPT,

DE || AB.

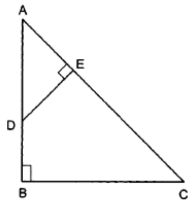
∴ If ∠A = ∠B and AD = BE then, DE || AB.

OR

Given: A triangle ABC in which $AB \perp BC$ and $DE \perp AC$.

To Prove: △ABC ~ △AED.

Proof: In △'s ABC and AED, we have



∠ABC = ∠AED = 90^o

∠BAC = ∠EAD (Each equal to ∠A]

Therefore, by AA-criterion of similarity, we obtain $\triangle ABC \sim \triangle AED$.

23. We have in given figure common tangents PQ and RS to two circles intersect at A. Since tangents drawn from an external points to a circle are equal.

$\therefore$ AP = AR
and AQ = AS
$\therefore$ AP + AQ = AR + AS [Adding]
$\Rightarrow$ PQ = RS
Hence proved.

24. $\frac{\cos 45^\circ}{\sec 30^\circ} + \frac{1}{\sec 60^\circ}$

$= \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}}} + \frac{1}{2}$

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{2}$

$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2}$

Rationalise the denominator,

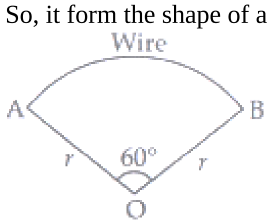
$= \frac{\sqrt{3}}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} + \frac{1}{2}$

$= \frac{\sqrt{6}}{4} + \frac{1}{2}$

25. i. The area of that part of the field in which the horse can graze if the length of the rope is 5cm
$= \frac{1}{4}\pi r^2 = \frac{1}{4} \times 3.14 \times (5)^2 = \frac{1}{4} \times 78.5 = 19.625\text{m}^2$
ii. The area of that part of the field in which the horse can graze if the length of the rope is 10 m
$= \frac{1}{4}\pi r^2 = \frac{1}{4} \times 3.14 \times (10)^2 = 78.5\text{m}^2$
$\therefore$ The increase in the grazing area
= 78.5 - 19.625 = 58.875cm^2

OR

Arc is a part of circle that makes 60° between radii at end points A and B of wire.
So, it form the shape of a sector.



r = ? l = 20 cm
$\therefore$ Length of arc $l = \frac{2\pi r\theta}{360^\circ}$
$\Rightarrow$ 20 cm = $\frac{2\times\pi\times r\times 60^\circ}{360^\circ}$
$\Rightarrow$ 2πr = 20 $\times$ 6
$\Rightarrow$ r = $\frac{120}{2\pi} = \frac{60}{\pi}$ cm
Hence, radius (r) = $\frac{60}{\pi}$ cm.

Section C

26. Numbers are of two types - prime and composite.
Prime numbers can be divided by 1 and only itself, whereas composite numbers have factors other than 1 and itself.
It can be observed that
7 × 11 × 13 + 13 = 13 × (7 × 11 + 1)
= 13 × (77 + 1)= 13 × 78= 13 ×13 × 6
The given expression has 6 and 13 as its factors.
Therefore, it is a composite number.
7 × 6 × 5 × 4 × 3 × 2 × 1 + 5
= 5 ×(7 × 6 × 4 × 3 × 2 × 1 + 1)
= 5 × (1008 + 1)= 5 ×1009
1009 cannot be factorized further
Therefore, the given expression has 5 and 1009 as its factors.
Hence, it is a composite number.

27. Let the given polynomial is p(x) = 4x^2 + 4x + 1
Since, α, β are zeroes of p(x),
$\therefore \alpha + \beta$= sum of zeroes = $\frac{-4}{4}$

Also, $\alpha.\beta$= Product of zeroes = $\alpha.\beta = \frac{1}{4}$

Now a quadratic polynomial whose zeroes are 2α and 2β

x^2 - (sum of zeroes)x + Product of zeroes

$= x^2 - (2\alpha + 2\beta)x + 2\alpha \times 2\beta$

$= x^2 - 2(\alpha + \beta)x + 4(\alpha\beta)$

$= x^2 - 2 \times (-1)x + 4 \times \frac{1}{4}$

= x^2 + 2x + 1

The quadratic polynomial whose zeroes are 2α and 2β is x^2 + 2x + 1

28. The given pair of linear equations

2x + 3y = 11 ...... (1)

2x - 4y = -24 .... (2)

From equation (1), 3y = 11 - 2x

$\Rightarrow \quad y = \frac{11-2x}{3}$

Substituting this value of y in equation (2), we get

$2x - 4\left(\frac{11-2x}{3}\right) = -24$

$\Rightarrow 6x - 44 + 8x = -72$

$\Rightarrow 14x - 44 = -72$

$\Rightarrow 14x = 44 - 72$

$\Rightarrow 14x = -28$

$\Rightarrow x = -\frac{28}{14} = -2$

Substituting this value of x in equation (3), we get

$y = \frac{11-2(-2)}{3} = \frac{11+4}{3} = \frac{15}{3} = 5$

Verification, Substituting x = -2 and y = 5, we find that both the equations (1) and (2) are satisfied as shown below:

$2x + 3y = 2(-2) + 3(5) = -4 + 15 = 11$

$2x - 4y = 2(-2) - 4(5) = -4 - 20 = -24$

This verifies the solution,

Now, y = axe + 3

$\Rightarrow 5 = m(-2) + 3$

$\Rightarrow -2m = 5 - 3$

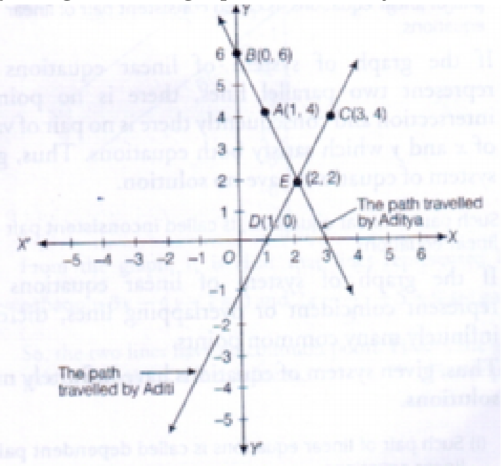
$\Rightarrow -2m = 2$

$\Rightarrow \text{m} = \frac{2}{-2} = -1$

OR

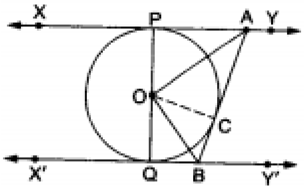
Let the given points be A(1,4) , B(0,6) , C(3,4) and D(1,0).

On plotting points A and B and joining them, we get the path travelled by Aditya. Similarly, on plotting points C and D and joining them, we get path travelled by Aditi.



It is clear from the graph that both of them cross each other at point E(2,2).

29. According to the question, XY and X 'Y ' are x two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersects XY at A and X 'Y ' at B.



In quad. APQB, we have

$\angle APO$ =90°

and $\angle BQO$ = 90° [∵ tangent at any point is perpendicular to the radius through the point of contact]

Now, $\angle APO + \angle BQO + \angle QBC + \angle PAC = 360^\circ$

$\Rightarrow \angle PAC + \angle QBC = 360^\circ - (\angle APO + \angle BQO) = 180^\circ$ ...(i)

We have,

$\angle CAO = \frac{1}{2}\angle PAC$

and $\angle CBO = \frac{1}{2}\angle QBC$ [∵ tangents from an external point are equally inclined to the line segment joining the centre to that point]

$\therefore \angle \text{CAO} + \angle \text{CBO} = \frac{1}{2}(\angle PAC + \angle QBC) = \frac{1}{2} \times 180^\circ = 90^\circ$ .....(ii)

In $\triangle AOB$, we have

$\angle CAO + \angle AOB + \angle CBO = 180^\circ$

$\Rightarrow 90^\circ + \angle AOB = 180^\circ$

$\Rightarrow \quad \angle AOB = 90^\circ$

30. LHS$= \left(1+\tan^2 A\right) + \left(1+\frac{1}{\tan^2 A}\right)$

$= \left(1+\tan^2 A\right) + \frac{(\tan^2 A+1)}{\tan^2 A}$

$= \sec^2 A + \frac{\sec^2 A}{\tan^2 A}$ $\left[\because 1+\tan^2 A = \sec^2 A\right]$

$= \frac{1}{\cos^2 A} + \frac{\frac{1}{\cos^2 A}}{\frac{\sin^2 A}{\cos^2 A}} \begin{bmatrix} \because \sec^2 A = \dfrac{1}{\cos^2 A} \\ \tan^2 A = \dfrac{\sin^2 A}{\cos^2 A} \end{bmatrix}$

$= \frac{1}{\cos^2 A} + \frac{1}{\cos^2 A} \times \frac{\cos^2 A}{\sin^2 A}$

$= \frac{1}{\cos^2 A} + \frac{1}{\sin^2 A}$

$= \frac{\sin^2 A+\cos^2 A}{\cos^2 A \sin^2 A}$

$= \frac{1}{\cos^2 A \sin^2 A}$ $\left[\because \sin^2 A + \cos^2 A = 1\right]$

$= \frac{1}{(1-\sin^2 A)\sin^2 A}$ $\left[\because \cos^2 A = 1 - \sin^2 A\right]$

$= \frac{1}{\sin^2 A - \sin^4 A}$

= RHS

Hence proved.

OR

L.H.S.

= sec A(1 - sin A)(sec A + tan A)

$=\frac{1}{cosA}$(1 - sin A)($\frac{1}{cosA}$ + $\frac{sinA}{cosA}$)

$=\frac{(1-sinA)}{cosA}(\frac{1+sinA}{cosA})$

$= \frac{(1-\sin A)(1+\sin A)}{\cos A \times \cos A}$

$= \frac{(1^2-\sin^2 A)}{\cos^2 A}$.[Since, (a - b) (a + b) = a^2 - b^2]

$= \frac{(1-\sin^2 A)}{\cos^2 A}$

$= \frac{cos^2 A}{cos^2 A}$

= 1

=RHS

Hence, proved.

31. i. Total outcomes = {1, 2, 4, 6, 8, 10}, n(S) = 6

favourable outcomes = {2, 6, 8, 10, 4}, n(f) = 5

$P = \frac{n(f)}{n(S)}$

$= \frac{5}{6}$

ii. Total case = 20 (Total no. of marbles)

Black marbles = 6

P(winning) = $\frac{n(\text{ Black marbles })}{n(\text{ Total marbles})}$

$= \frac{6}{20}$

P(winning) = $\frac{3}{10}$

Section D

32. Let the original list price be Rs x

$\therefore$ No. of books bought for Rs 300 = $\frac{300}{x}$

Reduced list price of the book = Rs (x - 5)

No. of books bought for Rs 300 = $\frac{300}{x-5}$

According to question,

$\frac{300}{x-5} - \frac{300}{x} = 5$

$\Rightarrow \frac{300x-300x+1500}{x^2-5x} = 5$

$\Rightarrow x^2 - 5x = 300 \Rightarrow x^2 - 5x - 300 = 0$

$\Rightarrow x^2 - 20x + 15x - 300 = 0$

$\Rightarrow (x-20)(x+15) = 0$

$\Rightarrow x = 20 \quad \text{or} \quad x = -15$

$\Rightarrow x = 20$

The negative sign is rejected.

Therefore x = 20

Therefore the original price list is Rs. 20

OR

Perimeter = 82 m

$\Rightarrow$ 2 (l +b) = 82 m

or, l + b = 41 m

Area = 400 m^2

$\Rightarrow$ l $\times$ b = 400 m^2

Let length be x m. Then,

breadth = (41- x) m

Now, $x(41-x) = 400$

$41x - x^2 = 400$

$x^2 - 41x + 400 = 0$

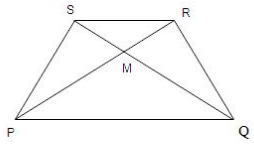
(x - 16)(x - 25) = 0

x = 16 or x = 25

Hence, if length = 16 m, then breadth = 25 m

or, if length = 25 m, then breadth = 16 m

33. Given : $\Delta PMS \sim \Delta QMR$ and $PQ || SR$.



To show PS = QR

$\because \triangle PMS \sim \triangle QMR$

$\therefore \frac{PS}{QR} = \frac{PM}{QM} = \frac{MS}{MR}$ ...(i)

[corresponding sides of similar triangles are proportional]

Now, consider $\triangle PMQ$ and $\triangle RMS$

In these triangles, we have

$\angle PMQ = \angle RMS$ [vertically opposite angles]

$\angle MPQ = \angle MRS$ [alternate angles]

$\because \triangle PMQ \sim \triangle RMS$ [AA criteria]

$\therefore \frac{PM}{RM} = \frac{MQ}{MS}$

[corresponding sides of similar triangles are proportional]

$\Rightarrow \frac{PM}{QM} = \frac{MR}{MS}$ ...(ii)

From Eq (i) and Eq (ii), we get

$\Rightarrow \frac{MS}{MR} = \frac{MR}{MS}$

$\Rightarrow MS^2 = MR^2$

$\Rightarrow MS = MR$

From Eq(i) , we get

$\therefore \frac{PS}{QR} = \frac{MS}{MR}$

$\frac{PS}{QR} = 1$

$\Rightarrow PS = QR$ Hence proved.

34. The Radius of the toy (r) = 3.5 cm

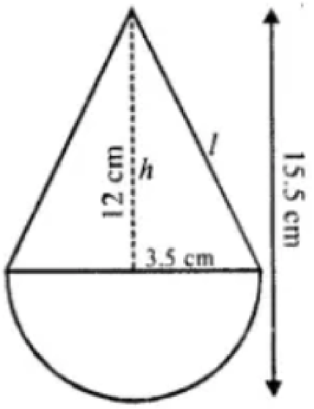
Total height of the toy = 15.5 cm

$\therefore$ Height of the conical part is = 15.5 - 3.5 = 12 cm

Slant height of the conical part (l)

$= \sqrt{r^2 + h^2} = \sqrt{(3.5)^2 + (12)^2}$

$= \sqrt{12.25 + 144} = \sqrt{156.25} = 12.5\text{cm}$



i. Now total surface area of the toy = curved surface area of conical part + curved surface area of hemipherical part

$= \pi rl + 2\pi r^2 = \pi r(l + 2r)$

$= \frac{22}{7} \times 3.5(12.5 + 2 \times 3.5)\ \text{cm}^2$

= 11 (12.5 + 7) = 11 $\times$ 19.5 cm^2

= 214.5 cm^2

ii. Volume of the toy $= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$

$= \frac{1}{3}\pi r^2(h + 2r)$

$= \frac{1}{3} \times \frac{22}{7}(3.5)^2(12 + 2 \times 3.5)\text{cm}^3$

$= \frac{1}{3} \times \frac{22}{7} \times 12.25(12 + 7)\text{cm}^3$

$= \frac{22}{3} \times 1.75 \times 19\text{cm}^3$

$= \frac{731.5}{3}$ = 243.83 cm^3

OR

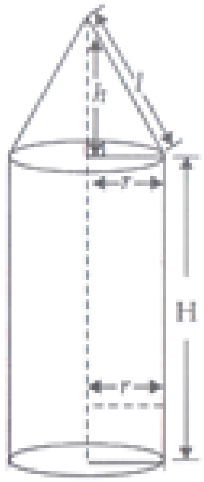
Cylinder	**Cone**
r = $\frac{6}{2}$ = 3 cm	r = 3 cm
H = 12 cm	l = 5 cm

For cone,

$\therefore$ $l^2 = r^2 + h^2$ + or $h^2 = l^2 - r^2$

$h^2 = 5^2 - 3^2 = 25 - 9 = 16$

$\Rightarrow$ h = $\sqrt{16}$ = 4 cm



Now, volume of rocket = Volume of cylinder + Volume of cone

$= \pi r^2 H + \frac{1}{3}\pi r^2 h = \pi r^2 \left[H + \frac{1}{3}h\right]$

= 3.14 × 3 × 3 $\left[12 + \frac{1}{3} \times 4\right]$

= 3.14 × 9 $\left[\frac{40}{3}\right]$ = 3.14 × 3 × 40 = 376.8 cm^3

∴ Volume of Rocket = 376.8 cm^3

Total surface area of rocket = Curved surface area of cylinder + Curved surface area of cone + Area of base of cylinder [As it is closed (Given)]

= 2πrH + πrl + πr^2 = πr [2H + l + r]

= 3.14 × 3[2 × 12 + 5 + 3]

= 3.14 × 3 × 32

= 301.44 cm^2

Hence, the surface area of rocket is 301.44 cm^2.

35. Since value of number of mangoes and number of boxes are large numerically. So we use step-deviation method

True Class Interval	**No. of boxes(f_i)**	**Class mark(x_i)**	$u_i = \frac{x_i - a}{h}$	**$f_i u_i$**
49.5-52.5	15	51	-2	-30
52.5-55.5	110	54	-1	-110
55.5-58.5	135	57	0	0
58.5-61.5	115	60	1	115
61.5-64.5	25	63	2	50
	$\sum f_i$ = 400			$\sum f_i u_i$ = 25

Let assumed mean (a) = 57,

h = 3 ,

$\therefore \bar{u} = \frac{\sum f_i u_i}{\sum f_i} = \frac{25}{400} = 0.0625$ (approx.)

Using formula, Mean $(\bar{x})$ = a + h$\bar{u}$

= 57 + 3 (0.0625

= 57 + 0.1875

= 57.1875

= 57.19 (approx)

Therefore, the mean number of mangoes is 57.19

Section E

36. **Read the text carefully and answer the questions:**

Your friend Varun wants to participate in a 200m race. He can currently run that distance in 51 seconds and with each day of practice it takes him 2 seconds less. He wants to do in 31 seconds.

(i) 51, 49, 47, ... 31 AP

d = -2

First 4 terms of AP are: 51, 49, 47, 45 ...

(ii) 51, 49, 47, ... 31 AP

d = -2

$t_n = a + (n - 1)d$

31 = 51 + (n - 1) (-2)

31 = 51 - 2n + 2

31 = 53 - 2n

31 - 53 = -2n

-22 = -2n

n = 11

i.e., he acheived his goal in 11 days.

OR

The given AP is

51, 49, 47, 45, 43, 41, 39, 37, 35, 33, 31, 29

$\therefore$ 30 is not in the AP.

(iii) 51, 49, 47, ... 31 AP

d = -2

$t_6 = a + (n - 1)d$

= 51 + (6 - 1)(-2)

= 51 + (-10)

= 41 sec

37. **Read the text carefully and answer the questions:**

The Chief Minister of Delhi launched the, 'Switch Delhi', an electric vehicle mass awareness campaign in the National Capital. The government has also issued tenders for setting up 100 charging stations across the city. Each station will have five charging points. For demo charging station is set up along a straight line and has charging points at $A\left(\frac{-7}{3}, 0\right)$, $B\left(0, \frac{7}{4}\right)$, C(3, 4), D(7, 7) and E(x, y). Also, the distance between C and E is 10 units.

(i) Here, CD = $\sqrt{(7-3)^2 + (7-4)^2}$

= $\sqrt{4^2 + 3^2}$ = 5 units

Also, it is given that CE = 10 units

Thus, DE = CE - CD = 10 - 5 = 5 units ($\because$ A, B, C, E are a line)

(ii) Since, CD = DE = 5 units

$\therefore$ D is the midpoint of CE.

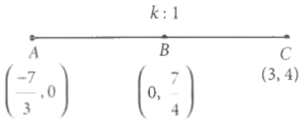
$\therefore \frac{x+3}{2} = 7$ and $\frac{y+4}{2} = 7$

$\Rightarrow$ x = 11 and y = 10 $\Rightarrow$ x + y = 21

OR

The points C, D and E are collinear.

(iii) Let B divides AC in the ratio k : 1, then



$\frac{7}{4} = \frac{4k+0}{k+1}$

$\Rightarrow$ 7k + 7 = 16 k

$\Rightarrow$ 7 = 9k

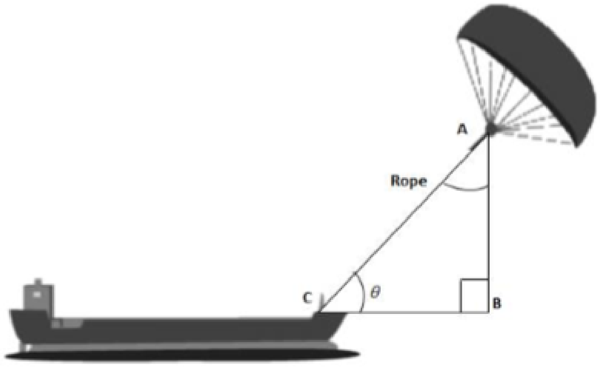
$\Rightarrow$ k = $\frac{7}{9}$

Thus, the required ratio is 7 : 9.

38. **Read the text carefully and answer the questions:**

Skysails is the genre of engineering science that uses extensive utilization of wind energy to move a vessel in the seawater. The 'Skysails' technology allows the towing kite to gain a height of anything between 100 metres - 300 metres. The sailing kite is made in such a way that it can be raised to its proper elevation and then brought back with the help of a 'telescopic mast' that enables the kite to be raised properly and effectively.

Based on the following figure related to sky sailing, answer the following questions:



(i) sin θ = cos(θ - 30^o)

cos (90^o - θ) = cos (θ - 30^o)

$\Rightarrow 90^\circ - \theta = \theta - 30^\circ$

$\Rightarrow \theta = 60^\circ$

(ii) $\frac{AB}{AC} = \sin 60^\circ$

$\therefore$ Length of rope, AC = $\frac{AB}{sin60^o}$ = $\frac{200}{\frac{\sqrt{3}}{2}}$ = $\frac{200 \times 2}{\sqrt{3}}$ = 230.94 m

OR

$\frac{AB}{AC} = \sin 30^\circ$

$\therefore$ Length of rope, AC = $\frac{AB}{sin30^o}$ = $\frac{150}{\frac{1}{2}}$ = 150 $\times$ 2 = 300 m

(iii) sin θ = cos(3θ - 30^o)

cos(90^o - θ) = cos(3θ - 30^o)

$\Rightarrow 90^\circ - \theta = 3\theta - 30^\circ \Rightarrow \theta = 30^\circ$